How to set up FMR sweeps based on static/conventional magnetometry data

Using SI units here: \( B \) [tesla] = \( \mu_0 (H+M) \)

For purposes of setting up initial FMR sweeps:

We can roughly estimate things easily by taking:

- \( \gamma/2\pi \sim 28 \text{ GHz/T} \) (pure spin gyromagnetic ratio)
- expressing \( \mu_0 H \) and \( \mu_0 M \) in units of tesla (1 T = 10 kOe)
- \( M_S \) can be calculated from \( M(H) \) loop from SQUID, knowing film thickness and area, and using conversion:
  \[ M_S \text{[tesla]} = M_S \text{[emu/cm3]} / 796 \]

Remember that the Kittel resonance condition all boils down to:

\[ f = \frac{\gamma}{2\pi} \text{ (magnetic field seen inside sample)} \]

and the discussion below is about determining that internal field which is the combination of:

- External applied field \( H \)
- Demag field \( H_0 = -N_d M \) where \( N_d \) is the demag factor (\( N = 1 \) for thin film is assumed below)
- In-plane anisotropy field \( H_k^{in} \) which is preference for moment to point along one direction in the plane of the film. This is usually very small in materials like CoFeB and NiFe.
- Perpendicular Uniaxial anisotropy field \( H_{k\perp} = 2K_u/\mu_0 M_S \)
  - In our convention here, \( H_{k\perp} > 0 \) tends to point moment out of plane
  - if \( H_{k\perp} \) is large enough, namely larger than the demagnetization field, then \( M_{\text{EFF}} = M_S - H_{k\perp} < 0 \), indicating we have PMA film
  - If \( H_{k\perp} = 0 \) this implies an easy plane anisotropy: the material wants the moment to lie in the plane, but this does not specify any particular direction in-plane (that is specified by \( H_k^{in} \))
  - One can also have a tilted anisotropy in which \( H_{k\perp} \) is not enough to completely bring the magnetization out of plane.

**Applied field Out of Plane (OOP) of film:**

\[ f_{\text{RES}} = \frac{\gamma \mu_0}{2\pi} (|H_{\text{RES}}| + H_K - M_S) = \frac{\gamma \mu_0}{2\pi} (|H_{\text{RES}}| - M_{\text{EFF}}) \]

So, an \( H_{k\perp} > 0 \) adds to the applied field, while the demag field \( H_0 = -M_S \) needs to be overcome in order to bring the moment out of plane and produce a strong resonance (\( f_{\text{RES}} > 0 \)).

- \( M_{\text{EFF}} \) is directly measured in SQUID \( M(H) \) data along hard-axis:
  - PMA film: hard axis is in-plane, so in-plane saturation field = \( H_{k\perp} - M_S = -M_{\text{EFF}} \)
  - IMA film: hard axis is out-of-plane, so OOP saturation field = \( M_S - H_{k\perp} = M_{\text{EFF}} \)
Applied Field In Plane (IP) of film:

\[
 f_{\text{RES}} = \frac{\gamma \mu_0}{2\pi} \sqrt{(M_{\text{EFF}} + H_{\text{IN}}^\text{in} + |H_{\text{RES}}|)(H_{\text{IN}}^\text{in} + |H_{\text{RES}}|)}
\]

\(H_{\text{IN}}^\text{in}:\) in-plane anisotropy field; i.e., anisotropy between directions X and Y in the plane of film. \(H_{\text{IN}}^\text{in} > 0\) means it is collinear with external field.

The first term in the square root is the field seen trying to get the moment out of plane, and again \(M_{\text{EFF}}\) appears since it is the total perpendicular anisotropy (competition between demag and uniaxial anisotropies).

The second term in the square root is for the in-plane direction that is perpendicular to the external field.

We recover the familiar textbook Kittel equations for OOP and IP (Kittel’s Intro to Solid State Physics, 5th Ed.) by ignoring \(H_{\text{IN}}^\perp\) and \(H_{\text{IN}}^\text{in}\).

Translating these equations to PhaseFMR software:

<table>
<thead>
<tr>
<th>PhaseFMR</th>
<th>here</th>
</tr>
</thead>
<tbody>
<tr>
<td>(M_S)</td>
<td>(M_{\text{EFF}})</td>
</tr>
<tr>
<td>IP (H_K)</td>
<td>(H_{\text{IN}}^\text{in})</td>
</tr>
<tr>
<td>OOP (H_K)</td>
<td>(H_{\text{IN}}^\perp)</td>
</tr>
</tbody>
</table>

Note that the PhaseFMR manual equations should make \(M_S \rightarrow M_{\text{EFF}}\) in the IP analysis, but otherwise agree with this document.
IP Example for IMA CoFeB film (50nm thick)

OOP saturation field $= 1.32$ T (from SQUID)

$\mu_0 M_{\text{eff}} = 1.267$ (from FMR) ($\gamma/2\pi = 29.3$ GHz/T taken from Shaw(2015)) (HK may be due to remanence?)
IP Example for PMA CoFeB film:

IP saturation field = 0.154 T (from SQUID)

\(\mu_0 M_{\text{eff}} = -0.15\ T\) (from FMR) (try forcing HK=0, changes Meff?)
OOP Example for PMA CoFeB film (same one as above):
IP saturation field = 0.154 T (from SQUID)
\(\mu_0 * M_{eff} = -0.124\) T (from FMR)

(Agreement not great: maybe we should not fix gamma/2pi to same value for thicker film?)
IP Example for Permalloy reference sample NiFe 10nm / Ta 5nm (sent in CryoFMR User Kit):
Measured in-plane in “hard” direction (Sharpie line is perp. to $H_{DC}$); fitted $H_k < 0$ validates this
OOP saturation field = 0.91 T (from SQUID – intersect lines from 0-0.6 T and 1.1-1.5T)
$\mu_o M_{eff} = 0.90$ T (from FMR)
OOP Example for Permalloy reference sample